

Quantifying Slopes with Digital Elevation Models of the Verdugo Hills, California: Effects of Resolution

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Abstract

Quantification of surface slope angles is valuable in a wide variety of earth sciences. Slopes measured from digital elevation models (DEM's) or other topographic data sets depend strongly on the length scale or window size used in the slope calculations. The spectrum of slope distributions as a function of length scale is related to the variation of relief as a function of scale, and may reflect the length scales of the processes acting to form the surface morphology. Small window sizes, approaching the DEM grid spacing, can yield reliable slope determinations if good quality data sets are available. There is a strong variation in the data quality among DEM's, with a 3-arc-second DEM poorly representing slopes in the Verdugo Hills, California area. The slopes measured from a standard 30-m DEM and high-resolution NASA/JPL TOPSAR DEM match each other well at length-scales greater than 30 m. Hillslope means for the rugged Verdugo Hills change from 15° to 26° as the calculation window size decreases from 270 m to 15 m.

Importance of Slope

The accurate measurement of the slope angle of the earth's surface is important in a variety of fields, including geomorphology, hydrology, civil engineering, natural hazard assessment, and ecology. Field measurements and theoretical studies have established that both hillslope and stream channel processes depend at least in part on the local

slope angle [e.g., Carson and Kirkby, 1972]. For example, the failure threshold of landslides (a sometimes catastrophic natural hazard) depends strongly on the surface slope and on the material properties of the soil and underlying rock [e.g., Carson and Kirkby, 1972; Dietrich *et al.*, 1993]. More gradual hillslope erosion is often modeled as a diffusive process with transport rate directly proportional to slope (so that the erosion rate is proportional to the surface curvature) [e. g., Culling, 1965], Channel initiation and transport theories typically include the local slope or channel gradient as a key parameter [Dietrich *et al.*, 1993]. The slope and aspect of an area can also have strong control on its microclimate, due to insolation and other effects.

Slope angles θ can be analyzed with different units: degrees or radians, $\sin \theta$, and $\tan \theta$. Whereas degrees are a common measure of angles, the component of gravitational force parallel to the slope ($\sin \theta$) and the gradient in m/m ($\tan \theta$) are more often used in equations of theoretical process models [e.g., Dietrich *et al.*, 1993]. Among the first studies to quantify slope-angle characteristics systematically were those by Strahler and colleagues in the 1950's [Strahler, 1950; 1954; 1956; Schumm, 1956], who defined hillslope angles either (1) as the maximum valley-side slope θ_{\max} , the steepest slope determined on selected contour orthogonal running from ridges to stream [Strahler, 1950], or (2) as the full surface-slope distribution measured systematically over an entire area [Strahler, 1956]. In tectonically deforming areas, maximum slopes (θ_{\max}) may represent the steepest slopes that can be maintained without slope failure in the rock or soil at the surface. The θ_{\max} method can also be practical for field measurements, because, in areas where a falling relative base-level is driving active incision, the maximum slope of valley sides is often located near the valley bottom, and a few dozen widely spaced determinations can be sufficient to characterize an area [Strahler, 1950].

The increasing availability of digital elevation models (DEM's) suggests that computerized slope analysis will become more common, because DEM's clearly provide an

opportunity to rapidly quantify topographic slopes over large areas. Key questions remain unanswered, however, about the appropriate applications and potential limitations associated with slope calculations from DEM's. For example, what is the effect of the size of the calculation "window" on slope determinations? What is the minimum window size that will yield reliable results? Do different types of DEM's from the same area produce the same slopes? To address these questions, we examine three different scales of DEM's for one of the classic areas of slope studies.

Digital Elevation Models

Digital elevation models are digital representations of the morphology of a surface, such as the elevation of the earth's land surface. DEM's use several representational methods, including contour lines or irregular and regular grids of elevation values, but this paper will only consider regular grids, DEM's with regular grids are efficiently represented as a two-dimensional matrix of elevation values with the horizontal location of each elevation determined by its place in the array and the grid geometry. The resolution of a DEM or image is often defined as the smallest resolvable separation between features [e.g., Forshaw, *et al.*, 1983], as such resolution is related to the accuracy (vertical or horizontal), In this paper, we use the term "slope resolution" for the shortest distance over which the slope of the surface can be reliably measured from a DEM. The grid spacing or horizontal precision of a DEM is often considerably smaller than the horizontal resolution and accuracy, and the vertical quantization (typically 1 m) is nearly always much smaller than the vertical elevation accuracy,

DEM's vary in both their grid spacing and resolution, depending on the data sources and production methods. The U.S. Geological Survey (USGS) distributes two DEM series, The "one-degree" DEM's have one-degree-square blocks on a grid of 3 arc-seconds (1200 x 1200 points per degree, roughly 90 m spacing), and cover all the

U.S. land area (although the grid spacing changes in Alaska) [USGS, 1993]. The U.S. Defense Mapping Agency (DMA) produced most of the U.S. one-degree DEM's at least 20 years ago by digitizing 1:250,000 topographic contour maps and then interpolating. This contour interpolation process often caused strong artifacts in the DEM visible in hypsometric curves and detailed shaded relief views as a series of false terraces with "treads" at the level of contours and "risers" between contours. The USGS also produces "30 m" DEM's for individual 7.5-minute quadrangles with a grid spacing of 30 m in the Universal Transverse Mercator (UTM) projection [USGS, 1993], with roughly half of the US covered so far. Most of the 7.5-minute DEM's are produced by several photogrammetric methods from aerial photographs, but some are derived from digitization of 1:24,000 contour maps. The artifacts or errors in these DEM's depend on the production procedure for each quadrangle. We use both types of USGS DEM' in this study,

The NASA/JPL airborne synthetic aperture radar (SAR) system has a special interferometric mode (TOPSAR) that produces high-resolution DEM's [Madsen *et al.*, 1995]. SAR interferometric systems have a certain level of thermal noise (approximately 13 dB SNR for the TOPSAR system) in the signal that corresponds to random errors in the resulting elevation measurements [Madsen *et al.*, 1995]. In the processing of SAR interferometry, there is a trade-off between horizontal and vertical resolutions depending on the amount of averaging or smoothing of adjacent values. Artifacts or noise in other DEM production techniques can also be filtered out to some extent by smoothing. More smoothing degrades the horizontal resolution, but increases the vertical accuracy by reducing the noise. The TOPSAR data used in this study was smoothed with a "boxcar" filter window 15-m wide, so the horizontal resolution is -15 m even though the grid spacing is 5 m. The vertical height accuracy (relative) is about 2 m with this degree of smoothing [S. Jansley personal communication, 1995].

Effects of Resolution

Hillslope angles can be efficiently measured from DEM's with regular grids [e.g., Evans, 1972]. Simple differencing of adjacent grid values to measure the slope is highly susceptible to noise or errors in the DEM, as would be expected for a derivative function. A more robust method of measuring slopes is to use a least-squares fit of a plane (or higher order polynomial) to the points within a "window" or square neighborhood and determine the slope of that plane (or linear component of a higher order fit). This technique has the effect of smoothing the slope values to a resolution approximately equal to the window size. We step the window across the array of elevation values and fit a plane to the data within each new window. The resulting slope array can be visualized as a slope map, or the slope values in a certain area can be collected into a histogram to show the slope distribution for that region. Normalizing the slope histograms to convert them to fractional area in each slope bin or class allows direct comparison of histograms for differently sized areas.

Slope measurements for a given landscape depend on at least two factors: the resolution and accuracy of the DEM analyzed and the slope-calculation window size. We will show below that the latter is usually the most important factor. The analysis window size should be chosen to be at least as large as the horizontal resolution of the DEM (or larger if the vertical accuracy is poor) to produce accurate slope measurements. The number of grid points or pixels within a window of a given size obviously depends on the grid spacing of the DEM. We define the window size as the distance between the first and last pixel included. For example, a 90-m window size will have 19 x 19 points on the 5-m spacing of TOPSAR, 4 x 4 pixels on the 30-m USGS DEM's, and only 2 x 2 points on the -90-m (3-arcsecond) grid DEM's. A larger number of points will increase the accuracy of the slope determination for a given window size, but smaller

window sizes will generate a more detailed approximation to the ground slopes for scales at which most hillslope processes act.

The optimum length-scale for measuring slopes depends on the geomorphic features or processes being studied. Strahler [1950] recommended a length of -30 m (100 ft) for determining slope in a “medium-texture” landscape, formed by typical hillslope processes. Characterizing the rapid evolution of very small-scale landforms (badlands developed in a few years on a clay deposit at Perth Amboy, N. J.) required a length scale of --30 cm (1 ft) [Strahler, 1950; Schumm, 1956]. Slopes measured over smaller than optimal lengths can be affected by individual plants, rocks, or other minor objects, as well as being strongly influenced by inaccuracies in individual data points.

We investigate the effects of both DEM resolution and window size by analyzing three different DEM's that cover the Verdugo Hills, a small range north of Burbank and Glendale, California and south of the San Gabriel Mountains (Fig. 1). We projected the TOPSAR and 3-arcsecond DEM's into standard UTM coordinates (zone 11 for the Burbank and Pasadena 7.5-minute quadrangles that cover the Verdugo Hills) to facilitate correlation of slope distributions for the same areas. Strahler [1950; 1954; 1956] studied hillslope angles of several small drainage basins of the Verdugo Hills in the field and from 1:24,000 scale topographic maps, enabling a comparison of the DEM slopes with the slopes calculated from more traditional techniques. He used -30-m lengths for slope measurements made both in the field and from maps [Strahler, 1950].

Results and Discussion

We created slope histograms with window sizes varying from 15 m to 270 m from the TOPSAR DEM over the area of the Verdugo Hills (Fig. 1). When the DEM source and resolution are fixed, the slope-calculation window size is a strong control on the measured slope distributions (Fig. 2). There was no significant change in slope histo-

grams with different histogram bin sizes, ranging from 0.10 to 10, although the histogram became much “noisier” with smaller bin sizes. The measured slopes are steeper and have a broader distribution when the window size decreases because there is less smoothing. Larger windows are more likely to include topographic features such as valley bottoms or ridge crests, and the opposing slopes of these features decrease the mean slope calculated in the window. For window sizes ranging from 270-by-270 m to 15-by-15 m, the mean slope increased from 15° to 26°, respectively, and the standard deviation of the entire distribution of slopes for each window size increased from 6° to 11° (Fig. 3). The mean slope values (measured in degrees) appear to extrapolate to a mean slope somewhere between 28° and 30° for a zero-length window (Fig. 3). This might be the mean slope of the Verdugo Hills if we had an infinitely detailed DEM, but a fractal model of topography would predict steeper and steeper slopes with decreasing window sizes.

To compare the effects of the differing slope resolutions and accuracy of the various DEM's, we generated slope histograms for the same window sizes (90 and 270 m) from each data set (Fig. 4). The 90-m window size is large enough to be accurate for the 30-m DEM (4 x 4 points per window), and the excellent match between the TOPSAR and 30-m DEM slope distributions is consistent with both DEM's being a good approximation to the actual hill slopes at that length scale. The observed slope distribution with the same 4 x 4 points-per-window (270-m) window size for the 3-arcsecond DEM does not match the nearly coincident 270-m-window slope distributions from the 30-m and TOPSAR DEM's (Fig 4). The large fraction of shallow (even 0–10, slopes is a reflection of the false terraces from contour interpolation in the 3-arcsecond DEM of this area, and the sharp drop-off of slopes steeper than 20° also underestimates the drop-off at 25° measured from the other DEM's. Thus, we conclude that the slope resolution of the 3-arcsecond DEM is considerably larger than 270 m in the Verdugo Hills. Clearly,

a mean slope derived from this 3-arcsecond data would underestimate the actual slopes at that wavelength,

We tested the slope resolution limit of the 30-m DEM by reducing the window size to 30 m (slopes calculated on 2 x 2 point windows) and comparing it to the same window size for the TOPSAR DEM (Figure 5) in the Verdugo Hills. Unlike the larger window sizes, the slope distribution for the 30-m DEM does not exactly match the TOPSAR DEM slopes with 30-m windows (7 x 7 points). The 30-m DEM is missing some of the steepest slopes at the 30-m window size. This suggests that the USGS 30-m DEM in this area has a slope resolution slightly larger than its grid spacing. This result may not apply to DEM's for other 7.5-minute quadrangles, because the methods used to produce the USGS 30-m DEM's vary [USGS, 1993],

We also compared the slopes calculated from the DEM's to those measured by the traditional field methods. Strahler [1950] measured valley-side maximum slopes (θ_{\max}) in a set of small drainage basins on the south side of the Verdugo Hills that he called Kline Canyon. (Note that the recent 7.5' Burbank 1:24,000 quadrangle (1988) shows the name Cabrini Canyon in part of the same area.) We formed a "local maximum" slope map from TOPSAR (15-m windows) for the Kline Canyon area (area marked on Fig. 1) and plotted it along with the θ_{\max} distributions from valleys with active incision ("corroded") and less active incision ("protected") measured in the field [Strahler, 1950; 1954](Figure 6). The local maximum slopes were calculated by taking the maximum slope within 105 x 105 m windows. The TOPSAR local maximum slopes include some shallower and steeper slopes, but the distribution matches reasonably well the distribution of maximum slopes measured on valley sides by Strahler [1950]. Thus, we infer that the slope distributions calculated with 15-m windows from the TOPSAR DEM approach the slopes that one would measure in the field at similar length scales.

Conclusions

We conclude that slopes measured at only one spatial scale cannot completely characterize a given landform. The variation of slopes over a spectrum of spatial scales can be measured with DEM's that have high resolution and accuracy and sufficient areal coverage. We compared the slope accuracy of different DEM's by calculating slopes with the same window size or spatial scale on each DEM. In the area of the Verdugo Hills, California, the USGS/DMA 3-arcsecond grid (or "1 degree") DEM is not accurate for slopes even with a large --270-m window size, while the slopes measured from the USGS 30-m grid (or "7.5 minute") DEM appear to be reasonably accurate down to a 30-m window size, when compared to the higher resolution NASA/JPL TOPSAR DEM. When hillslopes, rather than valley bottoms or ridge crests, are measured with a 15-m square window, they approach the slopes measured in the field using 100-foot-long linear transects.

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Figures

- Figure 1: Shaded relief map of the Verdugo Hills area. Black line shows polygon surrounding the Verdugo Hills used to form histograms of Figures 2–5. White line shows polygonal area around “Kline Canyon” used to form histogram of Figure 6.
- Figure 2: Slope histograms for the Verdugo Hills measured from the TOPSAR data with a series of different window sizes. Histograms in this figure are plotted by connecting the midpoints of the 1-degree bins for clarity.
- Figure 3: Mean, mode, and 90th percentile slopes from TOPSAR DEM plotted versus window size.

Figure 4: Comparison of slope histograms from three different data sources for 90- and 270-m window sizes. Midpoints of 1 -degree bins are shown.

Figure 5: Comparison of slope histograms from TOPSAR and 30-m USGS DEM with a 30-m window size. Stepped histograms with 1-degree bins.

Figure 6: Comparison of field measurements of valley-side maximum slopes [Strahler, 1950; 1954] with local maximum slopes from TOPSAR for Kline Canyon area (outlined on Figure 1). Local maximum slopes determined in 105-m windows from TOPSAR slopes measured 15-m window size. All histograms formed with 2-degree wide bins.

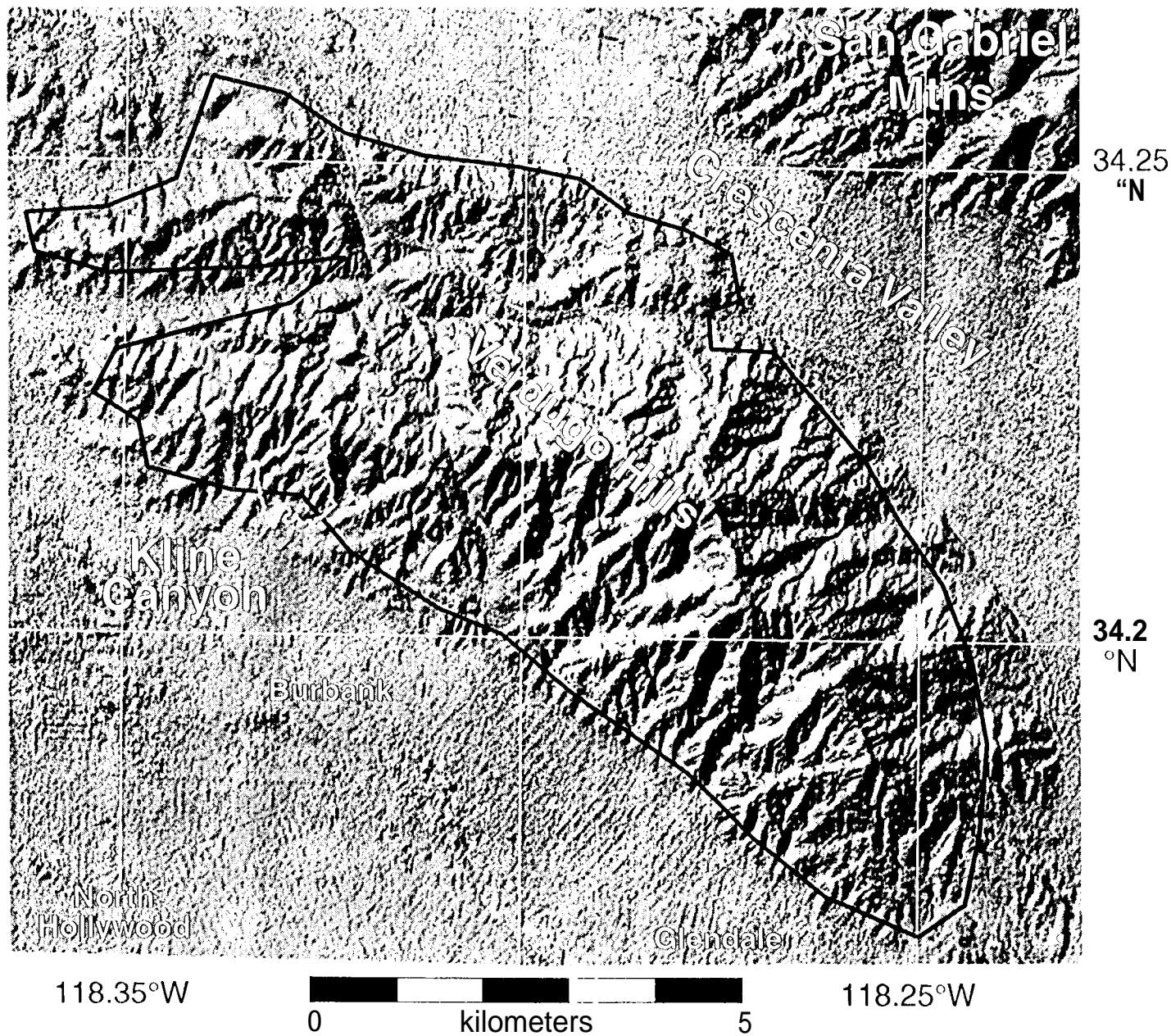


Figure 1

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Verdugo Hills TOPSAR slopes window size comparison

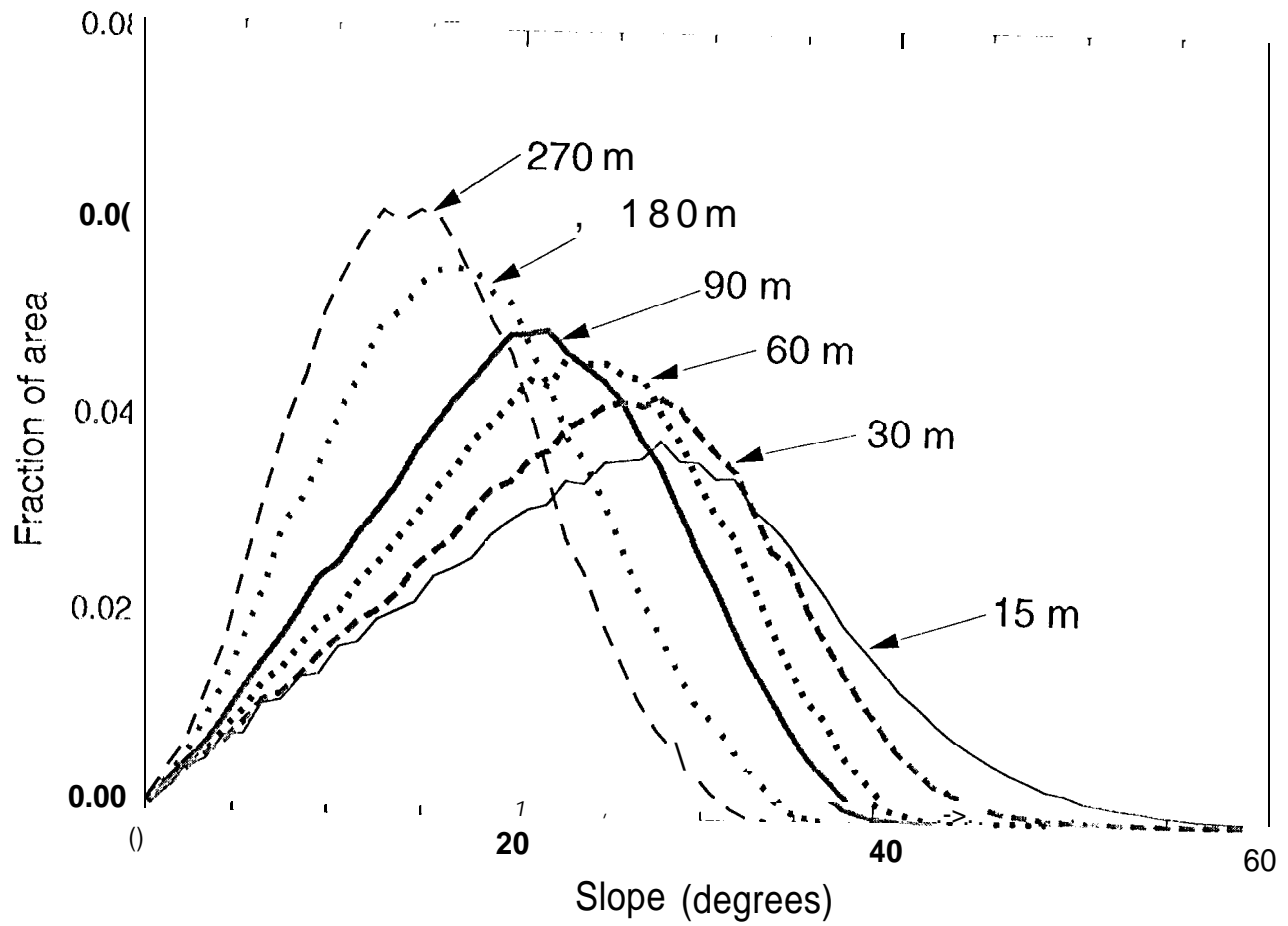


Figure 2
EJF 96/3/9

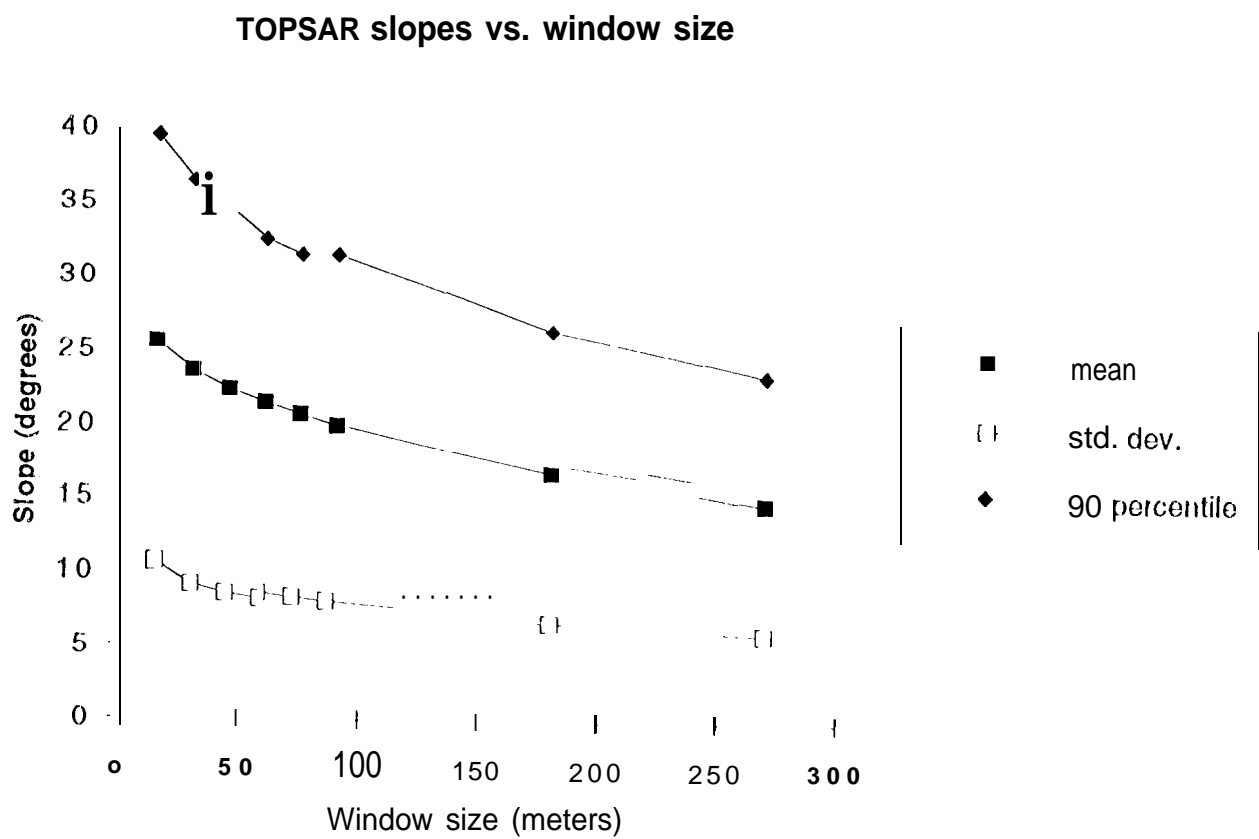


Figure 3
EJF 10/1 5/95

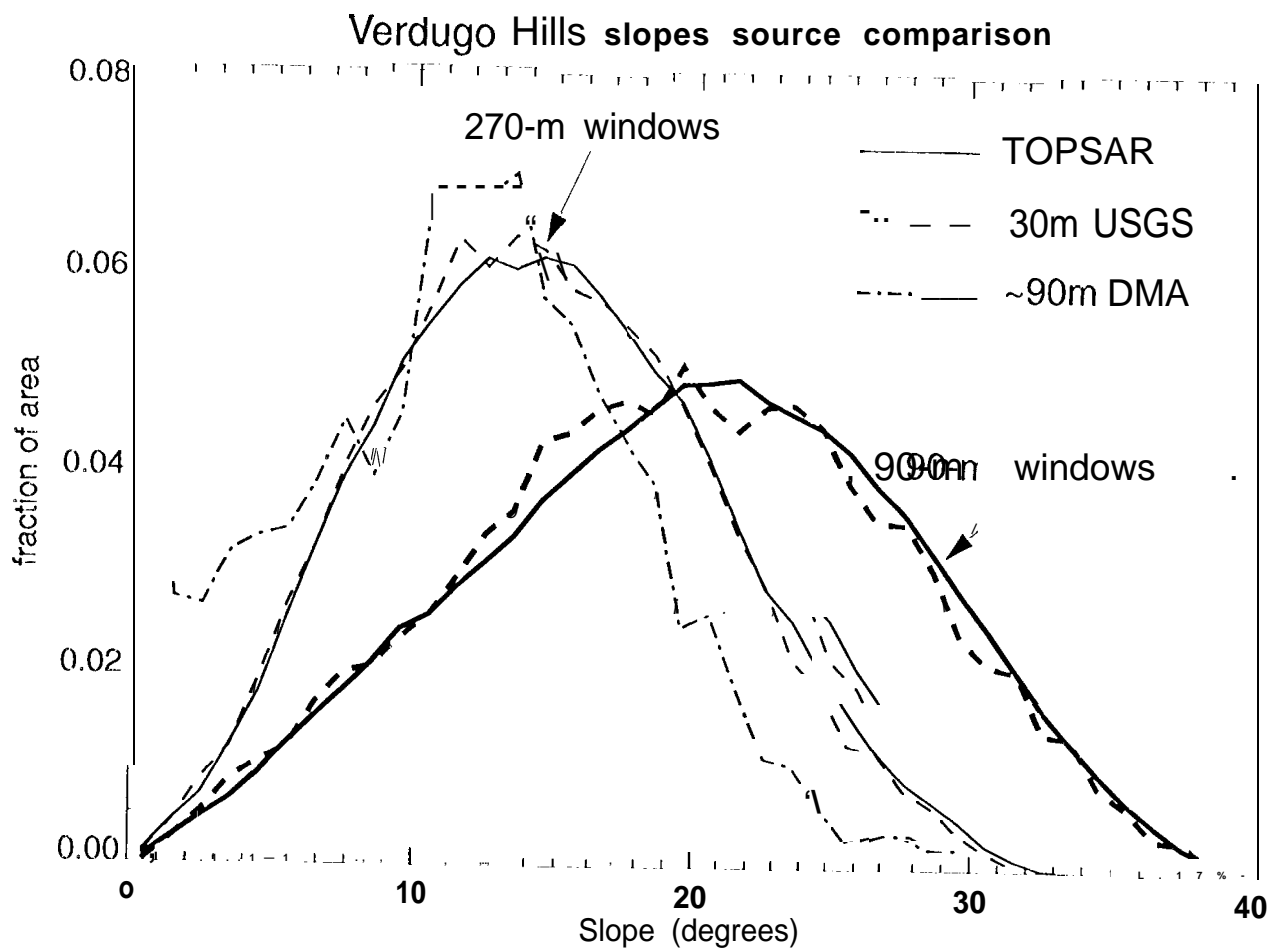


Figure 4
EJF 96/3/9

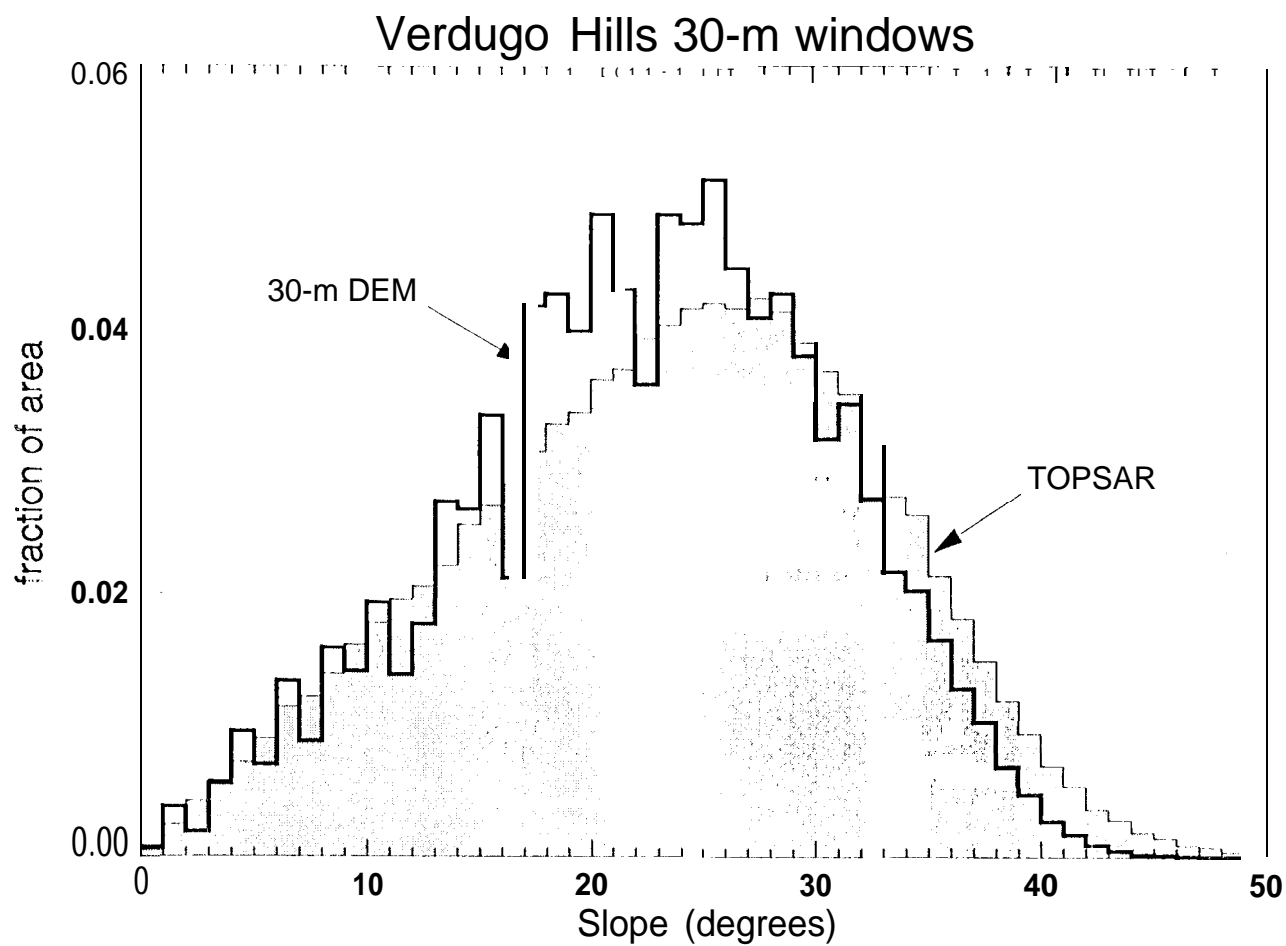


Figure 5
EJF 96/3/9

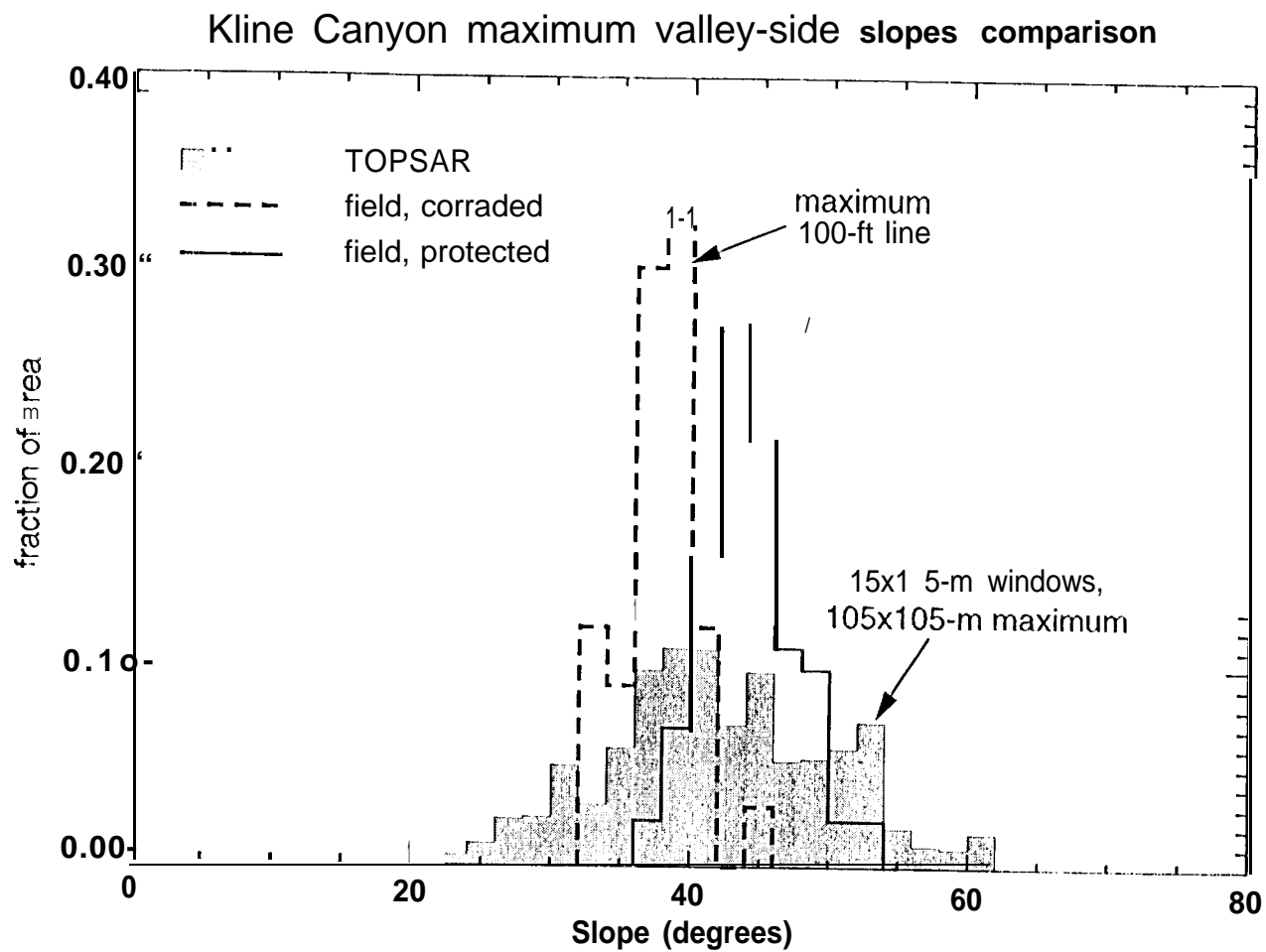


Figure 6
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